Bayesian parameter estimation with weak data and when combining evidence: the case of climate sensitivity

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Main areas to be discussed

- Why Objective not Subjective Bayes is needed for parameter estimation
- How correctly to combine evidence in the Objective Bayesian case

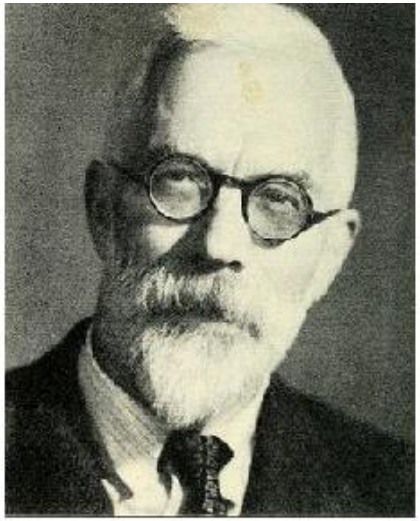
Probability is not a settled field

- Probabilistic inference has a troubled history
- Bitter disputes and personality clashes in past
- Deep disagreements over fundamental issues
- A few main belief sets and multiple variants
- In parts more like philosophy than mathematics
- Bayesian inference disdained for much of 20th C
- Subjective Bayes now strong: suits computers

Key figures in probability & statistics

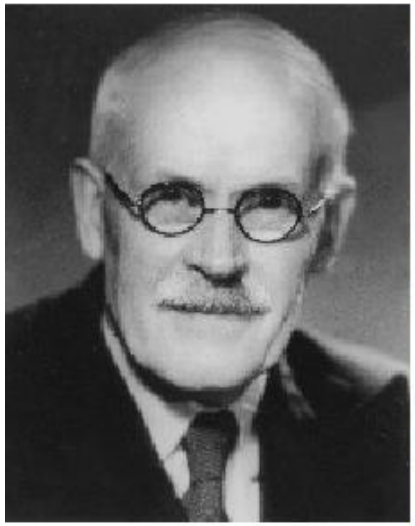
Bayes 1702–1761

Laplace 1749–1827



Fisher 1890-1962

Neyman 1894-1981



Jeffreys 1891-1989

de Finetti 1906-85

Savage 1917-71

Fraser 1925-

Bayesian estimation: continuous case

- Usual, Subjective method: apply Bayes theorem
- Likelihood: probability density for data at observed value *y*, expressed as a function of the parameter θ
- Prior: estimated PDF for θ given current evidence
- Posterior PDF= Likelihood x Prior PDF, normalised
- $p(\theta|y) = c\, p(y|\theta)\, p(\theta)$; c set so that $\int p(\theta|y)d\theta = 1$
- x%–y% range: posterior CDF credible interval; CrI

Does Bayes theorem apply here?

- Bayes theorem restates conditional probability $p(\theta|y)\ p(y) = p(y,\theta) = p(y|\theta)\ p(\theta)$; divide by $p(y)$
- Mathematically valid in the continuous case iff y and θ are Kolmogorov random variables (KRV)
- Bayes theorem gives mathematical probability iff parameter value is <u>random</u> with known prior PDF
- A fixed but unknown parameter is <u>not</u> a KRV
- Subjective Bayes provides coherent personal beliefs about parameters, **not** scientific validity

Objective Bayesian estimation

- Objective Bayes methods retain Bayesian form
- Objective Bayes uses ‘noninformative’ prior (NIP)
- NIP lets info. in data dominate; typically $\rightarrow$ CrI≈CI
- NIP is a weighting factor: no probability meaning

Noninformative priors

- Jeffreys prior: sqrt(|Fisher information matrix|)
- NIP => inference invariant on reparameterisation
- NIP *inter alia* converts probability between data and parameter spaces: includes a Jacobian factor
- NIP is usually simple in data space – often uniform
- Can use Bayes in data space and change variables to parameter space – even if lower dimensional

Subjective vs Objective ECS estimation

- Many ECS estimates use Subjective Bayes
- Subjective ~ Objective Bayes if data strong
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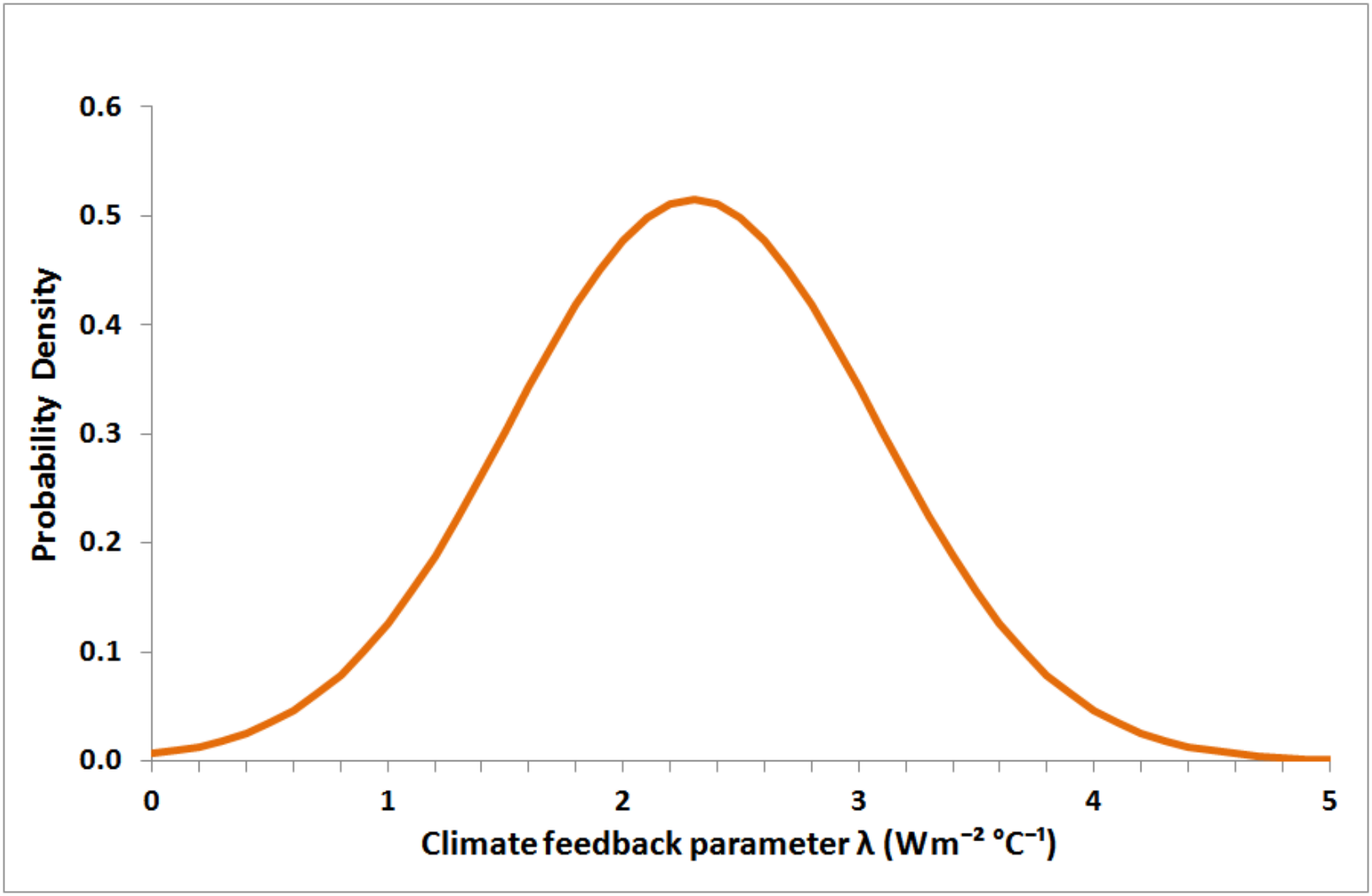
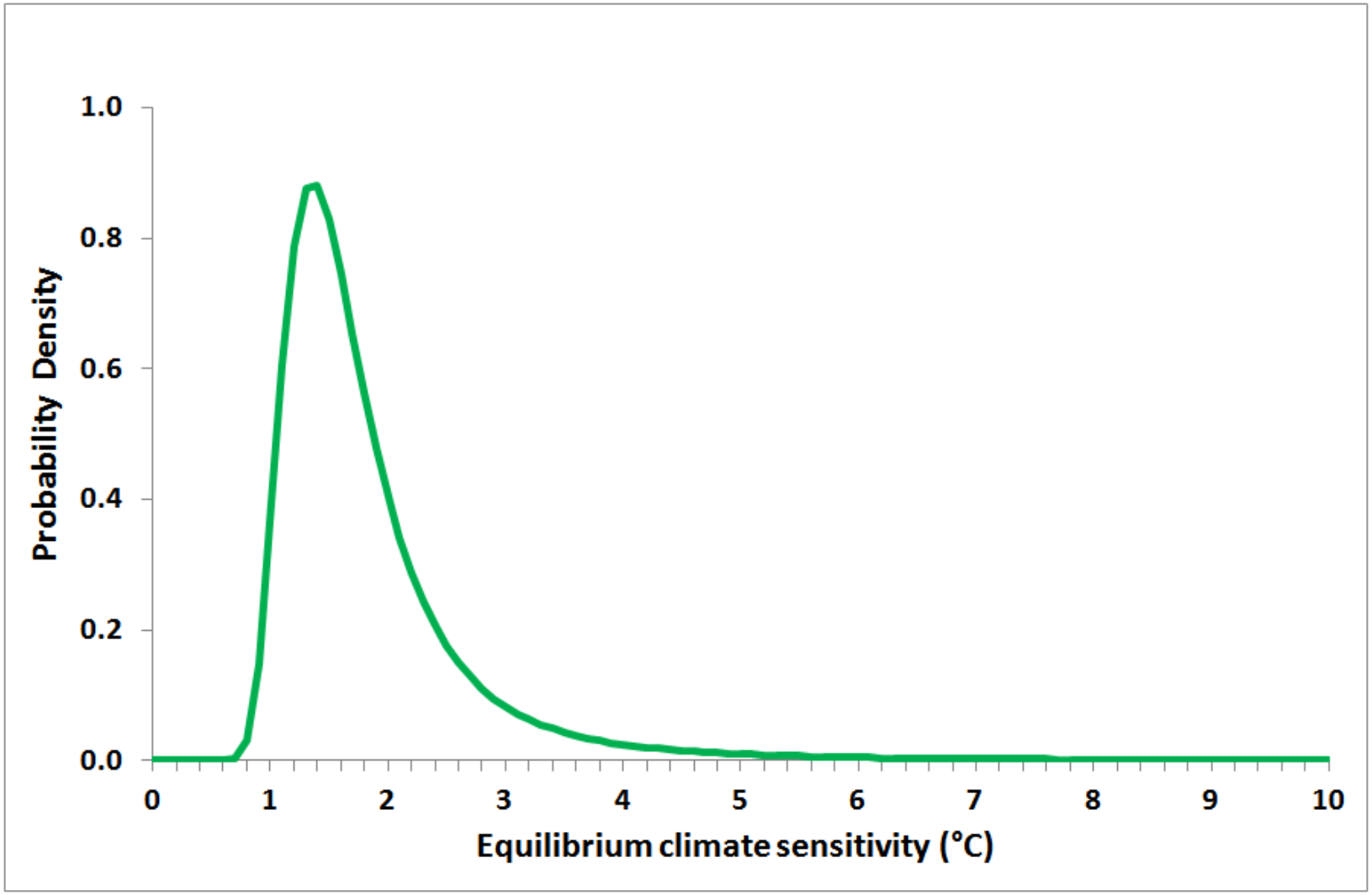
- ECS data too weak to dominate most priors
- Subjective Bayesian posterior PDFs for ECS generally don't reflect data error distributions
- Uniform ECS & K_v priors <u>greatly bias</u> estimates
- 'Expert' ECS prior usually dominates over data

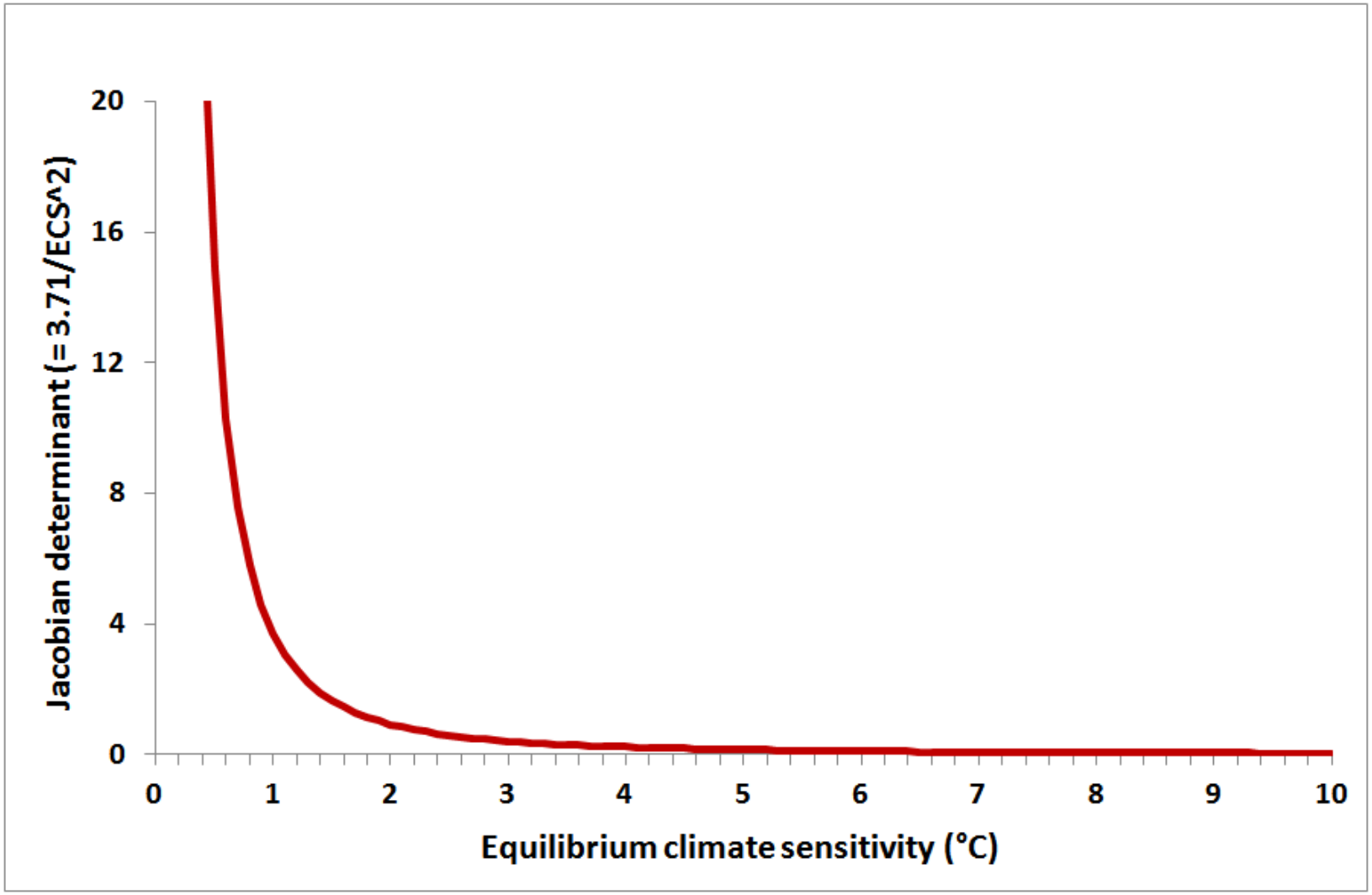
Estimating λ(∝1/ECS): Gaussian errors



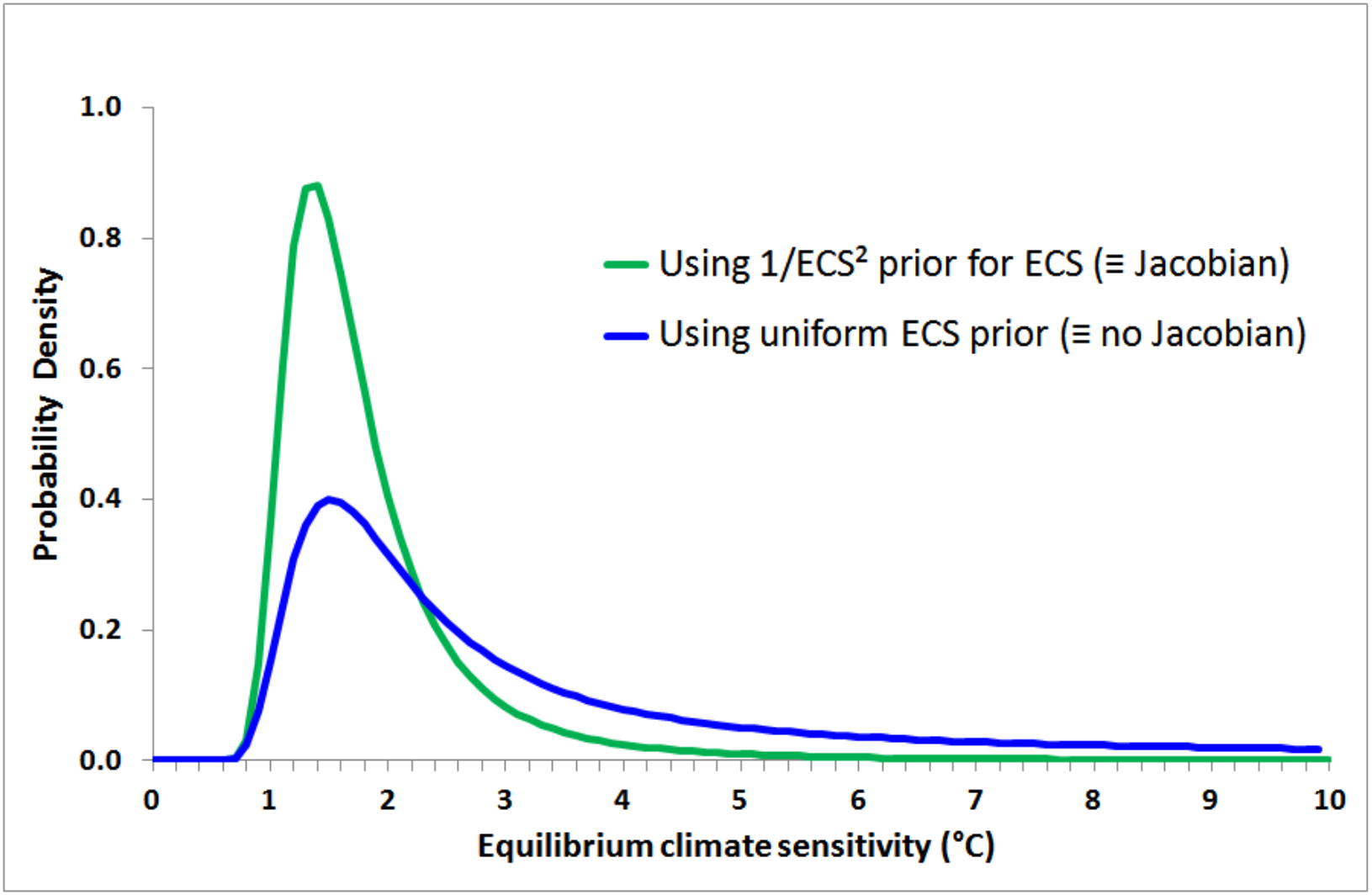
PDF converted (Jacobian) from λ to ECS



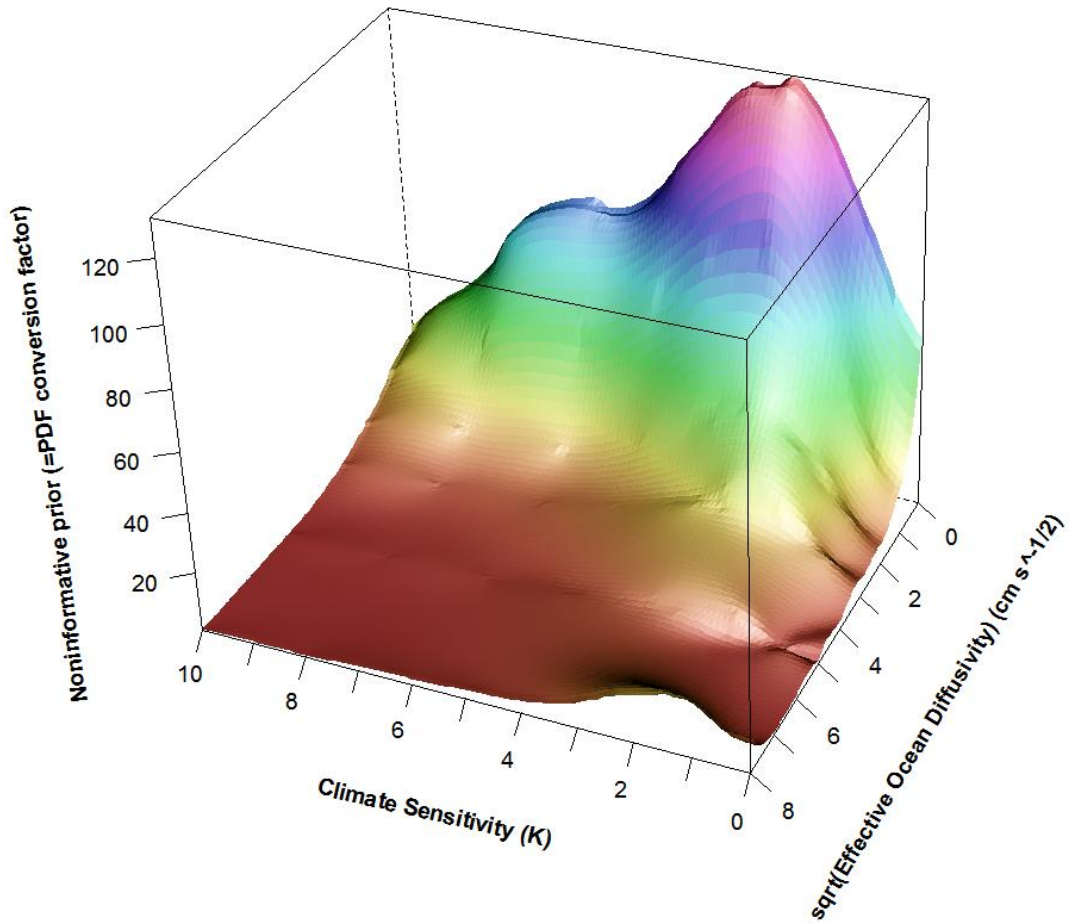
Jacobian for converting λ PDF to ECS



ECS PDF: effect of uniform prior vs NIP

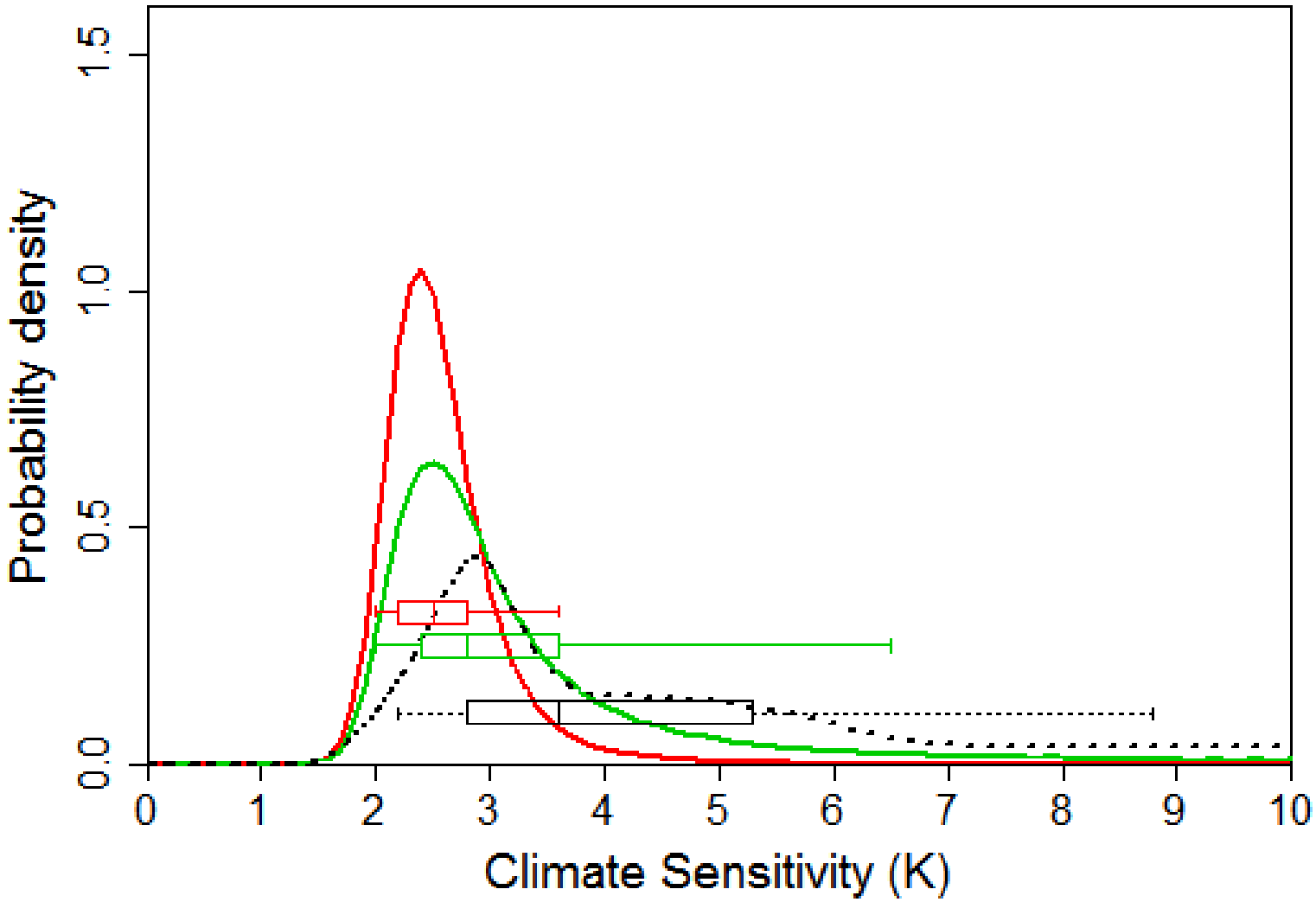


3D: Jeffreys prior for ECS, $\sqrt{K_V}$ & F_{aer}



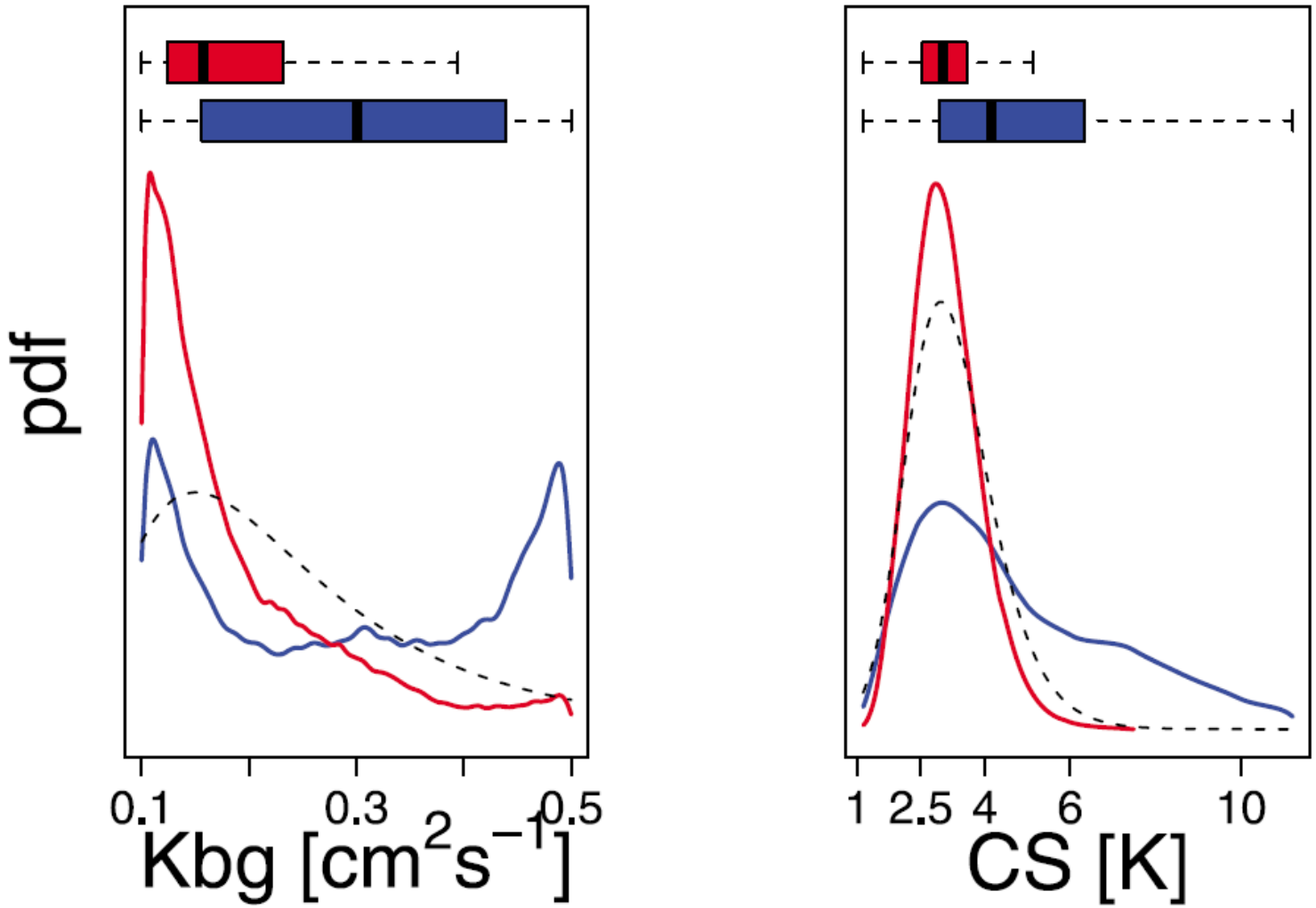
Lewis (2013) An objective Bayesian improved approach... J Clim

Noninformative v Uniform priors: 3D



Dominance of ‘expert’ priors

OLSON ET AL.: CLIMATE SENSITIVITY ESTIMATE



Combining ECS evidence

- Instrumental & paleo evidence ~ independent
- Standard Bayes method: combine by updating
- Final posterior= Π(likelihoods) x prior for 1st est.

Bayesian updating is unsatisfactory

- Objective Bayes + updating: order-dependence!
- NIPs likely to differ for each source of evidence
- Objective Bayes & Bayesian updating incompatible

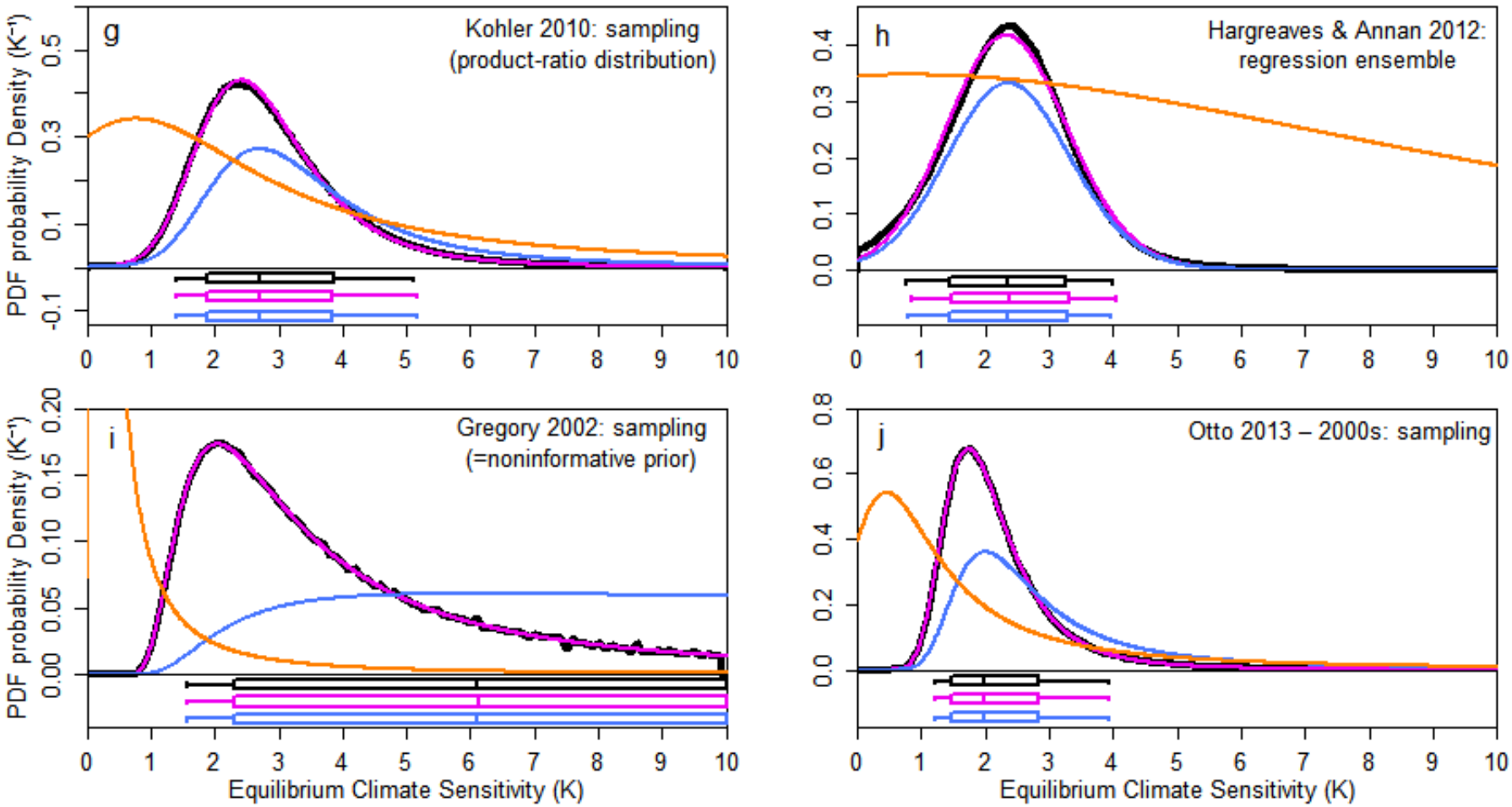
Avoiding Bayesian updating

- Solution: compute NIP for combined evidence
- Bayes on combined prior+combined likelihood
- To combine Jeffreys priors, add in quadrature
- Represent any prior info by data likelihood+NIP

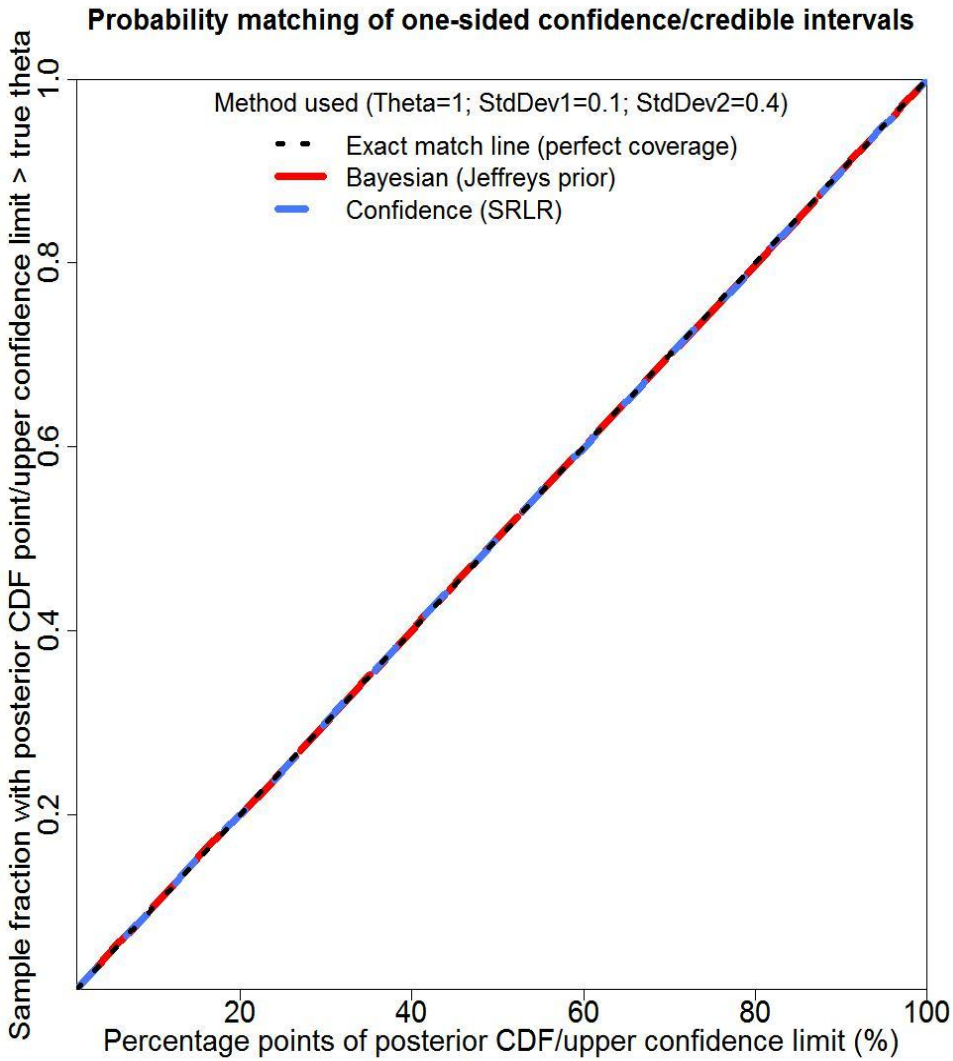
Objectively combining ECS evidence

- Bayes on combined prior+combined likelihood
- To combine Jeffreys priors, add in quadrature
- Apply to ECS estimation using parameterised distributions for which Jeffreys-like NIP known
- ECS ~$\propto$ ratio of two normals: ΔT / ($\Delta F - \Delta Q$)
- Good approximation to this distribution: RS93
- Method gives CrI=CI for combined ratio-normals

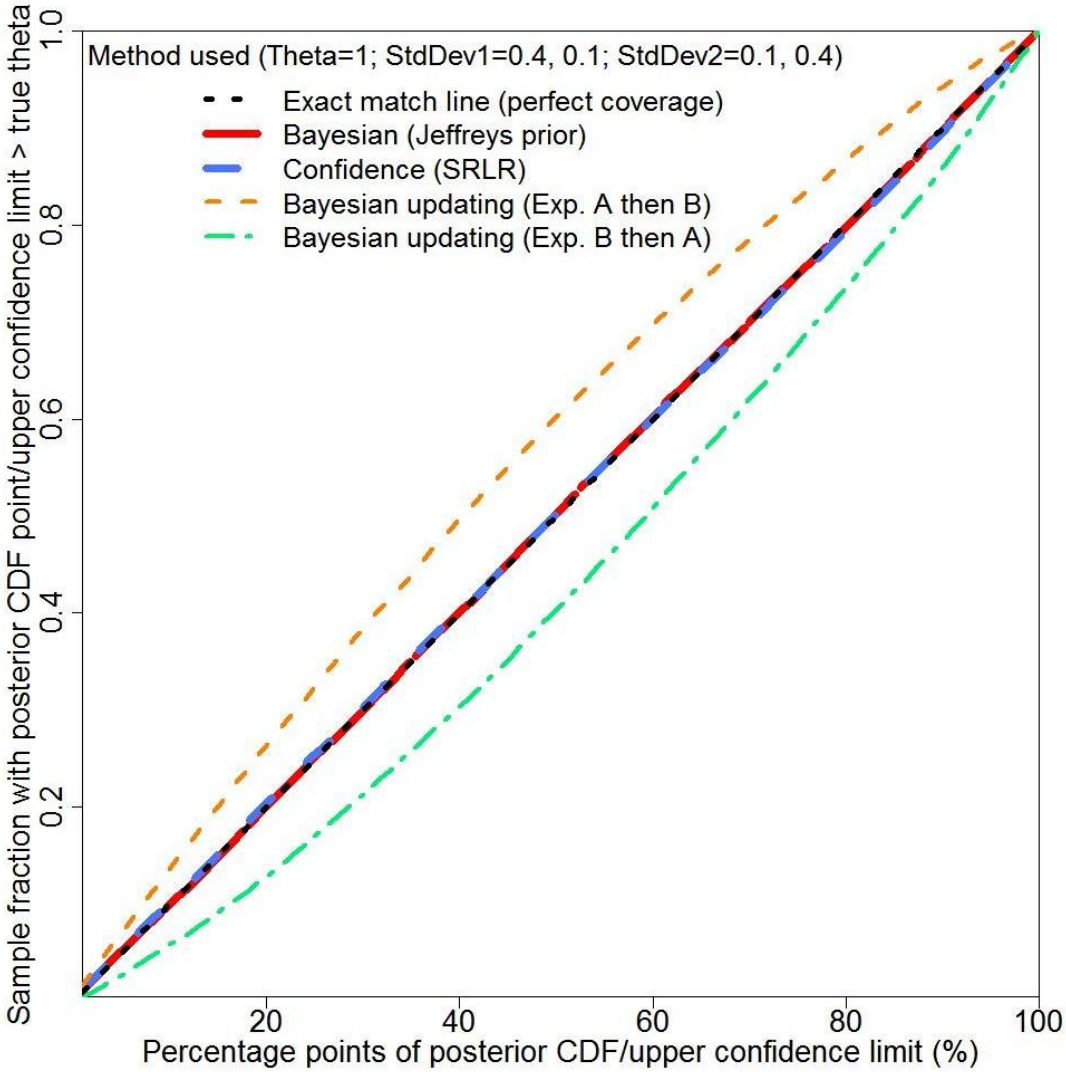
Ratio-normal approximation fits



Testing RS93-based probability matching



Testing probability matching: combination



Combining recent & paleo evidence

- Represent each ECS estimate by RS93 formula
- RS93 3-parameter formula fits ECS PDFs well
- Median and range suffice to uniquely specify

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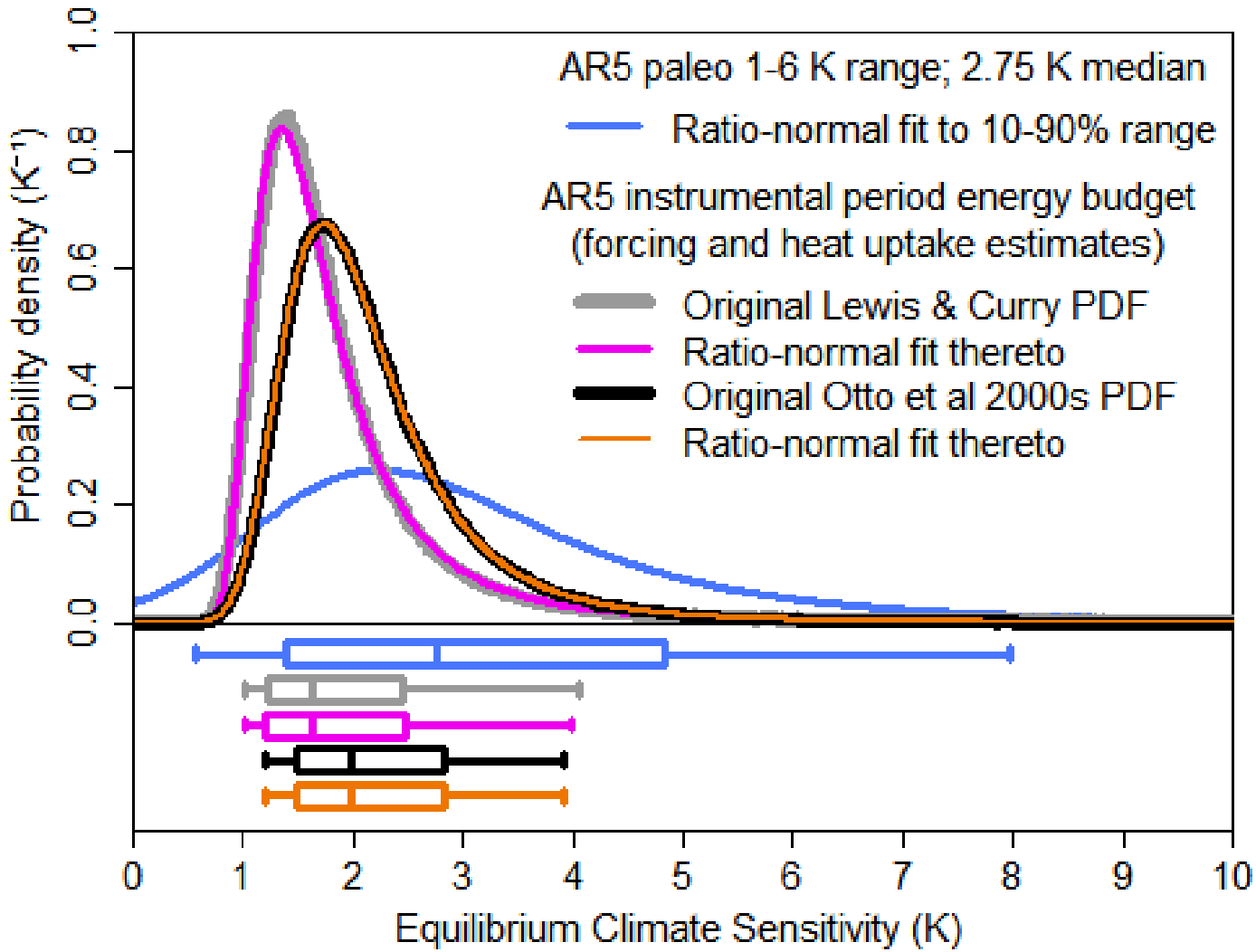
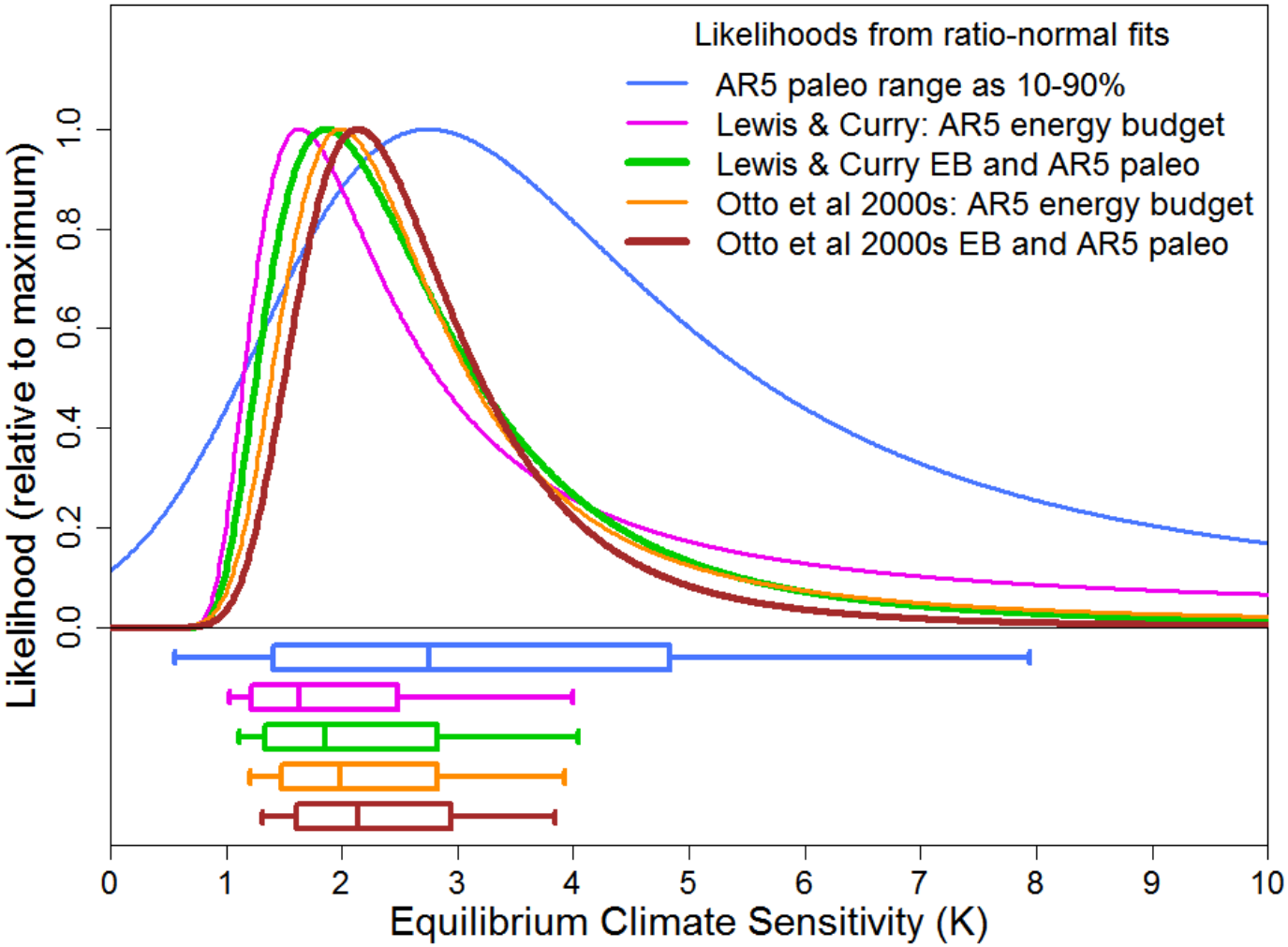
- Median and range suffice to uniquely specify
- Instrumental: Lewis & Curry 2015; Otto ea 2013
- Paleo range: 1–6 K per AR5, as exactly 10–90%
- Paleo median: 10 AR5 estimates; median 2.75 K

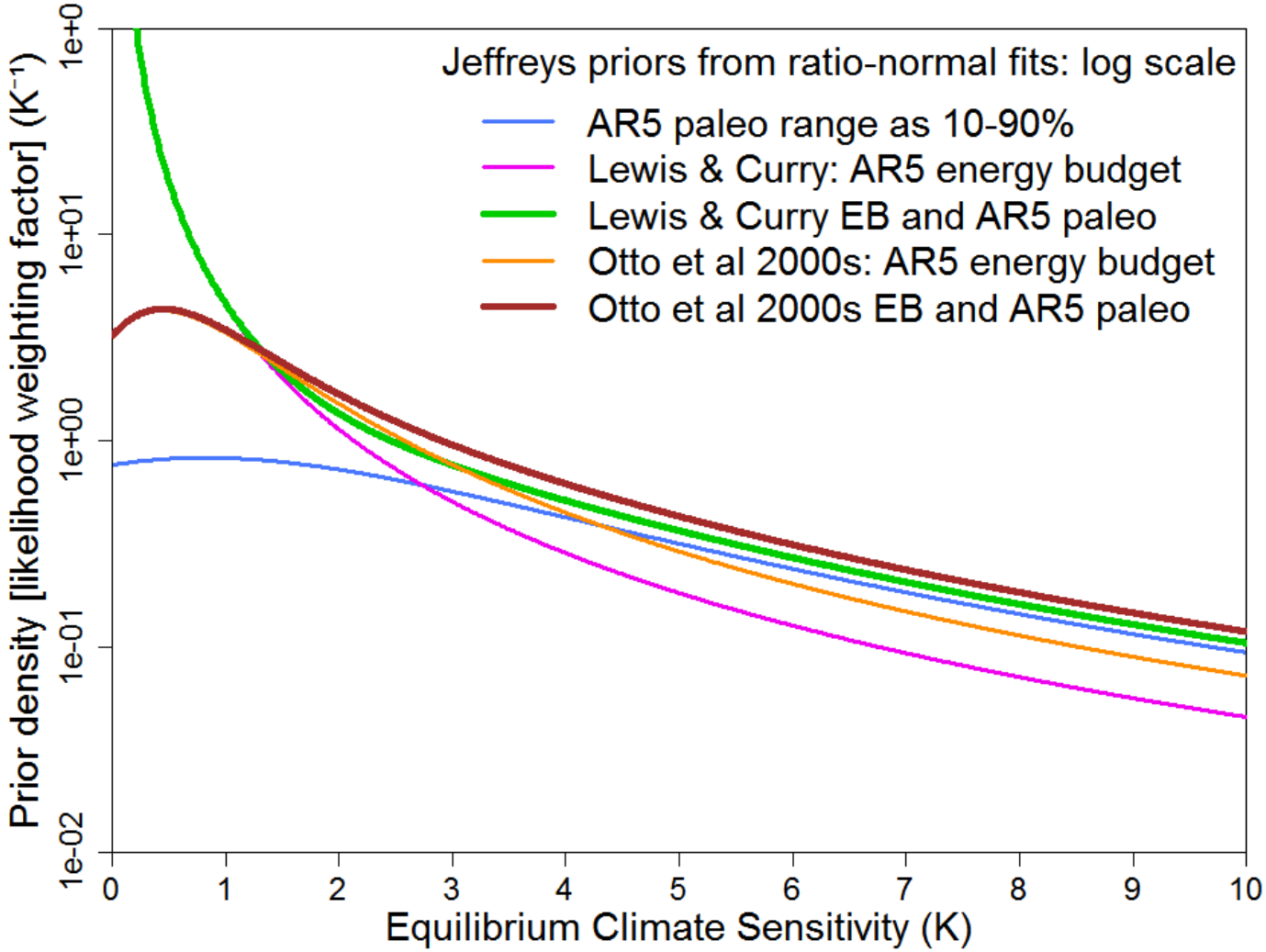
Ratio-normal fits to ECS estimates



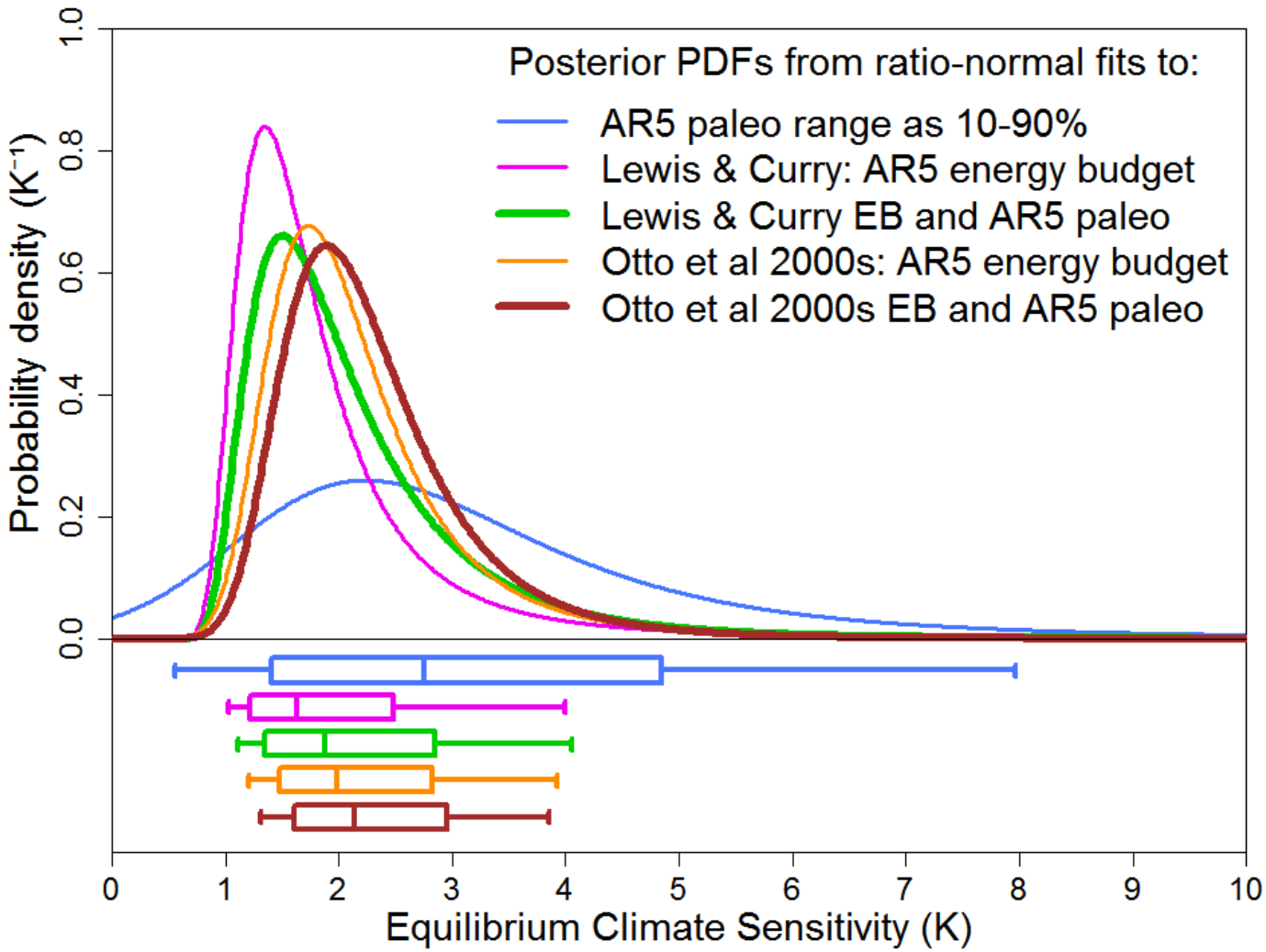
Original and combined likelihoods



Original+combined Jeffreys priors: log



Original and combined PDFs for ECS



Bayesian estimation: Conclusions

- Use Objective not Subjective Bayes if data weak
- Need to derive & compute noninformative prior
- Can infer ‘true’ data values & change variables
- Add Jeffreys priors[2] to combine info: don’t update
- Represent prior knowledge by data likelihood + NIP

Thank you for listening

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References

Lewis N, 2016. Combining independent Bayesian posteriors into a confidence distribution, with application to estimating climate sensitivity. JSPI (accepted)

Bernardo J, 2009. Modern Bayesian Inference: Foundations and Objective Methods.

Lewis N, 2013. An objective Bayesian improved approach for applying optimal fingerprint techniques to estimate climate sensitivity. J Clim, 26, 7414-29

Lewis N, 2013. Modification of Bayesian Updating where Continuous Parameters have Differing Relationships with New and Existing Data. *arXiv*:1308.2791

Lewis N, Curry JA, 2015. The implications for climate sensitivity of AR5 forcing and heat uptake estimates. Clim Dyn, *45, 1009–1023* DOI 10.1007/s00382-014-2342-y

Otto A et al, 2013. Energy budget constraints on climate response. Nat Geosci, 6, 415-6